

Introduction to Stochastic Processes, 2009

Assignment 1, solutions to first 3 exercises.

1. *Suppose that 5 percent of males and 0.25 percent of females are colourblind. If there are equal numbers of males and females in the population, what is the probability that a randomly selected colourblind person is male?*

Let $M = \{\text{person is male}\}$, so $\bar{M} = \{\text{person is female}\}$. Let $C = \{\text{person is colourblind}\}$. Thus $P(C|M) = .05$, $P(C|\bar{M}) = .0025$. If $P(M) = P(\bar{M}) = 0.5$, then by the law of total probability, $P(C) = (.5)(.05) + (.5)(.0025) = .02625$, and by Bayes' Theorem, $P(M|C) = \frac{P(C|M)P(M)}{P(C)} = \frac{(.05)(.5)}{.02625} = \frac{20}{21} \approx 0.9524$.

If $\frac{1}{6}$ of the colourblind persons are female, what is the proportion of females in the whole population?

Let $p = P(\bar{M}) =$ proportion of population that is female.

Then $P(C) = (1 - p)(.05) + p(.0025) = .05 - .0475p$, while

$$\begin{aligned}P(\bar{M}|C) &= \frac{P(C|\bar{M})P(\bar{M})}{P(C)} \\ \Rightarrow \frac{1}{6} &= \frac{.0025p}{.05 - .0475p} \\ \Rightarrow .05 - .0475p &= .015p \\ \Rightarrow .05 &= .0625p \\ \Rightarrow p &= .8\end{aligned}$$

2. *Suppose that E and F are mutually exclusive events of a random experiment, and that independent trials of this experiment are performed until either E or F occurs. Show that E occurs before F with probability*

$$\frac{P(E)}{P(E) + P(F)}.$$

If E occurs before F then there must be k trials whose outcome is in neither E nor F , followed by a trial whose outcome is in E , where $k \in \{0, 1, 2, \dots\}$. The probability of this is

$$\sum_{k=0}^{\infty} (1 - P(E) - P(F))^k P(E) = \frac{P(E)}{1 - (1 - P(E) - P(F))} = \frac{P(E)}{P(E) + P(F)}.$$

3. *Customers enter a supermarket at random intervals, at the average rate of 1 every 2 minutes.*

Here, “random” means Poisson process, that is, interarrival times are exponentially distributed (with mean 2 minutes) and there is no memory.

(a) What is the probability that no one enters the supermarket between 12:00 pm and 12:05 pm?

$$P(0 \text{ arrivals in 5 minutes}) = \frac{2.5^0}{0!} e^{-2.5} \approx .0821$$

(b) What is the probability that 4 customers enter during a 5 minute period?

$$P(4 \text{ arrivals in 5 minutes}) = \frac{2.5^4}{4!} e^{-2.5} \approx .1336$$

(c) If a customer has just entered, what is the probability that the next customer arrives and enters more than 2 minutes from now?

$$P(0 \text{ arrivals in 2 minutes}) = \frac{1^0}{0!} e^{-1} \approx .3679$$